SOME PROBLEMS OF THE FLOW OF A VISCOELASTIC MEDIUM IN TUBES OF CONSTANT SECTION

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The change of the pressure drop along the channel of a tube during flow of a viscoelastic medium is investigated.

The article is devoted to a determination of the regularities of the pressure drop along a channel of a tube during flow of a viscoelastic medium. The objects of investigation were two specimens of polyisobutylene grade P-20, differing in viscosity which corresponded to the shear stress τ extrapolated to zero. For the first specimen $\eta_{\tau \to 0} = 4.1 \cdot 10^5$ and for the second $\eta_{\tau \to 0}$ $2.4 \cdot 10^6$ Nsec/m^2 at 25°C. The specific weight of the specimens was 910 kg/m^3.

The pressure at the wall was measured by the strain gauge method similar to that done earlier by a number of investigators [1-3].

In conducting the experiments we use a high-pressure capillary viscosimeter [4] with a set of round and rectangular capillaries. In treating the experimental data the calculation results were obtained in the form of the values of stresses τ and shear velocities $\dot{\gamma}$ reduced to the conditions of deformation at the tube walls. For this purpose we used the conventional methods of treating experimental data [5] obtained during flow in tubes of constant section. The device for determining the distribution of stresses in a rectangular tube is shown schematically in Fig. 1. It represents a constant-pressure capillary viscosimeter. The pressure drop in the tube is created from a nitrogen cylinder 1. The pressure regulator 2 maintains a constant pressure in bomb 4. The pressure in the bomb is monitored by standard manometers 3 connected into a common control panel. The pressure of the test medium 8 filling the cylinder (bomb) 4 is transmitted through piston 5. The inside diameter of the bomb is 0.035 m and the length is 0.4 m. The piston was absent when conducting a series of experiments.

The test medium flowed through a plane rectangular channel of a tube 6. The height of the tube channel was H = 0.0055, width W = 0.0218, and length L = 0.149 m. The tube was made of polymethyl methacrylate plates, 0.007 m thick. Five grooves with a diameter of 0.018 m were milled on one of the plates (as indicated in Fig. 1) so that the wall thickness of the channel directly in the grooves was reduced along the channel from the entrance to the exit from 0.001 to 0.0003 m, respectively. Wire strain gauges 7 were cemented in the grooves on the wall parallel to the channel of the tube. The distance of the center of the first gauge from the tube entrance section was 0.025 m and the distance between all subsequent centers of the gauges was 0.026 m. The base of the wire strain gauges was 0.005 × 0.004 m.

The gauges were connected to a balancing bridge circuit supplied by a stabilized direct current. The change of the resistance of the gauges during deflection of the tube walls under the effect of the pressure of the test medium were recorded on an ÉPP-09 recording potentiometer. The gauges were calibrated under static conditions. In so doing the channel exit was closed, the bomb and tube were filled with vaseline oil, and constant pressures, monitored by manometers 3, were set. The curves of the change of the gauge readings as a function of the pressure delivered to the bomb during filling of the bomb and tube with polyisobutylene with a closed exit of the tube coincided with the calibration curves.

The volume flow rate Q was calculated on the basis of determining the weight of the test medium flowing from the tube in fixed time intervals.

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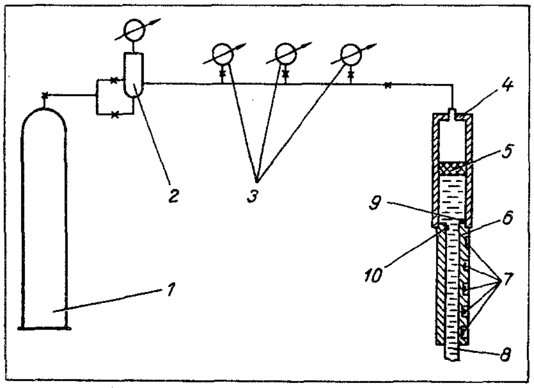

Fig. 1

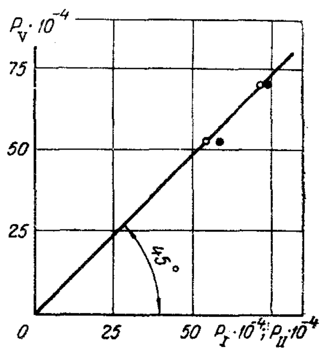

Fig. 2

Fig. 1. Diagram of the device for measuring the change of pressure over the length of the rectangular tube.

Fig. 2. Pressures P_I and P_{II} at points corresponding to positions 9 and 10 in Fig. 1 as a function of the pressure P_V of the gas in the viscosimeter bomb at shear stresses $\tau = 0.81 \cdot 10^4$ and $\tau = 1.06 \cdot 10^4$ N/m² in the second specimen of polyisobutylene.

The shear velocity at the wall was determined by the equation [5]

$$\dot{\gamma} = \pm \frac{6Q}{WH^2} \text{ [sec}^{-1}] \tag{1}$$

and the shear stress

$$\tau = 9.81 \cdot 10^4 \frac{H}{2} \cdot \frac{dP}{dl} \text{ [N/m}^2], \tag{2}$$

where dp/dl is the pressure gradient for a steady flow regime of the test medium in the tube.

The change of pressure immediately next to the entrance section of the tube channel was determined in the following way. The plane tube channel and viscosimeter bomb (Fig. 1) were filled with specimen 8 to a height of 0.011 m before the channel entrance (piston 5 was absent). At the entrance to the tube channel the pressures were measured on the end face (9) and at a distance of 0.0002 m from the plane of the tube end (10). Figure 2 presents the results of measuring the pressures at the entrance of the tube channel corresponding to positions 9 and 10. The straight line running at an angle of 45° to the coordinate axes corresponds to equality of the pressure P_V created in the viscosimeter bomb and measured at points denoted in Fig. 1 by positions 9 and 10. In Fig. 2 the corresponding pressures at points 9 and 10 are denoted as P_I and P_{II} and are plotted as closed and open circles. The experiments were carried out at shear stresses $\tau = 0.81 \cdot 10^4$ and $\tau = 1.06 \cdot 10^4$ N/m². Under the indicated conditions elastic deformations in the test specimen did not develop owing to the absence of a difference of P_I and P_{II} within the measurement limits.

It is clear from the aforesaid that if elastic deformation did not develop in the second specimen up to the entrance section of the tube channel at $1.06 \cdot 10^4$ N/m² and height of filling the test material in the bomb h = 0.01 m, then for τ and H_V less than the indicated values the elastic deformations and normal stresses in the entrance section of the tube channel will be equal to zero.

We will estimate the measurement errors made during this study.

The experimental data were treated on the basis of the calibration curves. In graphing the calibration curves, as when conducting the experiment, we measured the pressure and weight of the material flowing from the tube in definite time intervals.

The pressure was set according to a class 0.5 standard manometer and registered by strain gauges with recording on the ÉPP-09. The manometer readings were taken with a greater accuracy than their recording on the ÉPP-09. We recorded the values of the pressure $(P - P_{22})$ orthogonal to the shear planes with a maximum absolute error of ± 0.5 of the scale division of the ÉPP-09 recording potentiometer, which corresponds to reading errors to the gauge $\Delta(P - P_{22})$ placed near the entrance section of the tube channel

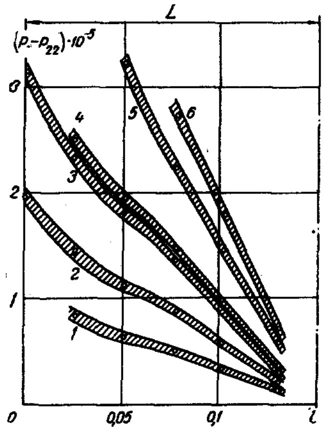

Fig. 3

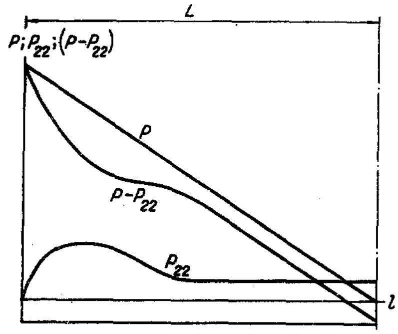

Fig. 4

Fig. 3. Distribution of pressure $(P-P_{22})$ at wall along the tube channel. Curves 1-6 for shear rates at the wall of 0.0073, 0.0115, 0.091, 0.02, 0.0374, and 0.0469 sec^{-1}, respectively. Curves 2 and 3 were obtained in the absence of a piston in the visosimeter bomb and height of filling the specimen $H_V < 0.01$ m.

Fig. 4. Diagram of the comparison of the values of P, P_{22}, and $(P-P_{22})$. L is the length of the tube channel.

of $\pm$ 0.06 atm $\simeq 6 \cdot 10$ N/m^2 and near the exit section of $\pm$ 0.03 atm $\simeq 3 \cdot 10^4$ N/m^2. The absolute errors of the pressure measurements at other points of the tube channel are between the indicated values, decreasing about in proportion to the distance from the entrance of the tube channel. In Fig. 3 the region of absolute errors of measuring $(P-P_{22})$ is hatched and within it lie the true values of $(P-P_{22})$. The absolute error or measuring $(P-P_{22})$ can be expressed in segments of the length of the tube channel Δl on which a pressure drop equal to $\Delta(P-P_{22})$, i.e., $\Delta l = \Delta(P-P_{22})/d(P-P_{22})/dl$, occurs. The accuracy of measuring $(P-P_{22})$ was determined only by the accuracy of the recording on the potentiometer, since the only variable parameter necessary for calculating $(P-P_{22})$, viz., the pressure in the viscosimeter bomb, was set with a much greater accuracy.

The relative errors, according to Fig. 3, at small pressures $(P-P_{22})$ reach appreciable values (about $\pm$ 25%), but at large pressures $(P-P_{22})$ amount to only about $\pm$ 1.5%.

Experimental Results. The distribution of the pressures of the test medium on the tube walls over the length of the plane channel during flow of polyisobutylene with different shear velocities through it is given in Fig. 3. The data for the first specimen are presented in Fig. 3. The pressure drop in the bomb as a consequence of the friction of the piston on its wall and flow of the test medium in the bomb, and also at the start of the tube up to the first gauge, can reach 67% of the pressure set in the bomb. In this case the indicated pressure losses decrease with an increase of the strain rate, which can be traced from the data in Table 1 (third column), where the measurement results only for the first specimen of polyisobutylene are given.

Under conditions in which the change of pressures was determined in the entrance section, i.e., with filling of the viscosimeter bomb to a height of 0.01 m, we determine the distribution of pressures in the tube channel during flow of the first specimen of polyisobutylene at shear rates of 0.0115 and 0.019 sec^{-1} (Fig. 3, curves 3 and 3). Curve 2 coincides fully with the curve obtained for the same shear rate but with filling of the viscosimeter bomb to a height of 0.11 m in the presence of a piston, although in the latter case the pressure losses outside the tube channel reach appreciable values m ~ 60% (Table 1, experiment Nos. 2 and 3), whereas in the first case they are practically equal to zero. This indicates that not only with filling to a height of 0.011 m but also to 0.11 m the elastic deformations in the entrance section of the tube channel are equal to zero, as also $P_{11} = P_{22} = 0$.

The most marked change of pressure $(P-P_{22})$ occurred at the entrance of the tube channel, where the normal stresses P_{11} and P_{22} were equal to zero. Under these conditions the structure of the material for

TABLE 1. Parameters Determining the Flow of a Specimen of P-20 Polyisobutylene in a Rectangular Tube Channel

№	No. of curve in Fig. 2	P, atm	$\dot{\gamma}$, sec^{-1}	$\frac{\partial P}{\partial l}$, atm	Measurements of pressure $(P - P_{22})$, atm, at points located from the entrance, m					Pressure losses, % in viscosimeter bomb	Shear stress at wall $\tau \cdot 10^{-4}$ for steady flow, N/m^2
					0,0025	0,0051	0,0077	0,0103	0,0129		
1	*1*	2,53	0,0073	0,087	0,83	0,62	0,52	0,3	0,08	67	0,187
2	*2*	3,44	0,0115	0,142	1,4	1,11	0,85	0,56	0,18	60	0,306
3	*2*	1,97	0,0115	0,142	1,45	1,11	0,83	0,55	0,16	0	0,306
4	*3*	3,14	0,019	0,214	2,34	1,79	1,33	0,86	0,28	0	0,46
5	*4*	4,97	0,02	0,228	2,5	1,96	1,46	0,92	0,33	49,6	0,49
6	*5*	7,05	0,0374	0,352	—	3,22	2,26	1,45	0,56	—	0,756
7	*6*	8,37	0,0469	0,417	—	—	2,73	1,77	0,68	—	0,895

all practical purposes still did not experience damage. The viscosity $\eta_{\dot{\gamma} \to 0}$ was determined by the following equations, which are a particular case of the notation of the notation of the equation [5] of motion of our problem for the entrance section of the tube channel:

$$\frac{dP}{dl} = d(P - P_{11})/dl, \quad \frac{d(P - P_{22})}{dl} = \frac{d\tau}{dh},$$

since $P_{11} = P_{22} = 0$ when $1 \to 0$, i.e., for the entrance section, we can find the shear stress at the wall $\tau_{\dot{\gamma} \to 0}$ and the apparent viscosity corresponding to this stress $\eta_{\dot{\gamma} \to 0} = \tau_{\gamma \to 0}/\dot{\gamma}$.

For the first specimen of polyisobutylene the pressure gradients at the channel entrance for shear rates 0.0115 and 0.019 sec^{-1} (Fig. 3, curves 2 and 3) are respectively $2.15 \cdot 10^4$ and $3.57 \cdot 10^4$ N/m^2. The viscosity $\eta_{\dot{\gamma} \to 0}$ according to these data is equal to $4.1 \cdot 10^5$ $Nsec/m^2$ in both cases. The viscosity determined for this same specimen on extrapolating the shear stress to zero is equal to $4.3 \cdot 10^5$ $Nsec/m^2$ (extrapolation according to the data in Table 1).

For the second specimen of polyisobutylene the viscosity $\eta_{\dot{\gamma} \to 0}$ determined according to the pressure drop $d(P-P_{22})/dl$ in the entrance section of the tube channel is equal to $2.5 \cdot 10^6$ $Nsec/m^2$; the determination of $\eta_{\tau \to 0}$ on extrapolation to a zero value of the shear stress gives a value of $\eta_{\tau \to 0}$ equal to $2.25 \cdot 10^6$ Nsec $/m^2$.

From the comparison indicated above we see that under appropriate conditions the method of calculating the viscosity $\eta_{\tau \to 0}$ on the basis of the pressure gradient $d(P-P_{22})/dl$ at the channel entrance can be used practically for determining the maximum viscosity corresponding to its value when $\tau \to 0$.

In the region of the tube channel where we can assume that the pressure gradient remains constant, the increment of the elastic energy is equal to zero and the normal stresses remain unchanged: $dP_{22}/dl = dP_{11}/dl = 0$, whence follows $d(P-P_{11})/dl = d(P-P_{22})/dl = dP/dl$. This enables us to calculate in this region on the basis of the experimentally determined values of $d(P-P_{22})/dl$ the shear stresses by means of the usual equations [5] for a steady flow regime of the test material.

The following scheme of comparing the data obtained on rotary and capillary instruments is of interest: the normal stresses during shear with an increase of strain, just as the shear stresses, reach a maximum and then decrease, as is shown schematically in the graph for P_{22} (Fig. 4). The presence of this variation of normal stresses, for example, for the first and second specimens of polyisobutylene at shear rates corresponding to those used in determining the relations presented in Fig. 3, follows from the fact that at the same shear rates, for example, for the first specimen, the elastic deformations as a function of total deformation correspond to the graph for P_{22} (Fig. 4). The elastic deformations were determined by direct measurements on a rotary instrument [7]. If we assume that the most probable case of linearity of the hydrostatic pressure over the length of the tube channel is realized (the graph for P_{22}, Fig. 4), then the function $(P-P_{22}) = f(1)$ corresponds to the graph given for this function in Fig. 4, which corresponds fully to the experimental curves presented in Fig. 3.

In the case of determining not only $(P-P_{22}) = f(1)$ but also $(P-P_{11}) = (1)$ the method of capillary viscosimetry probably permits taking a large part of the measurements which are taken with the use of rotary instruments.

NOTATION

P_V	is the pressure in the viscosimeter bomb;
P_I and P_{II}	are the pressure in the end plane at the channel entrance and in a direction orthogonal to the shear plane at a distance of 0.002 m from the entrance section of the tube channel;
P_{11} and P_{22}	are the normal stresses directed along the tube channel and orthogonal to the shear plane;
P	is the hydrostatic pressure;
$(P-P_{22})$	is the pressure in a direction orthogonal to the shear plane;
$\Delta(P-P_{22})$	are the errors of measuring pressure $(P-P_{22})$;
H, W, L,	are the height, width, and length of tube channel;
H_V	is the height of column of test material in viscosimeter bomb;
h, l	are the directions orthogonal to the shear plane and along tube channel;
Q	is the volume flow rate;
$\dot{\gamma}$	is the shear rate;
$\eta_{\tau \to 0}$ and $\eta_{\dot{\gamma} \to 0}$	are the viscosities corresponding to shear stress and deformation extrapolated to zero values;
τ	is the shear stress.

LITERATURE CITED

1. F. Schultz-Grunow and H. Weymann, Kolloid-Z., 131, 61-66 (1953).
2. F. Schultz-Grunow, VDI Zeitschrift, 97, No. 14, 409-416 (1955).
3. R. V. Torner, Inzh.-Fiz. Zh, 3, No. 6, 23-28 (1960).
4. G. V. Vinogradov and N. V. Prozorovskaya, Plasticheskie Massy, No. 5 (1964).
5. J. M. McKelvey, Plastics Processing, John Wiley and Sons, New York, (1962).
6. I. M. Belkin, G. V. Vinogradov, and A. I. Leonov, Rotary Instruments [in Russian], Mashinostroenie (1968).
7. A. S. Morozov, Inventor's Certificate 211145, Byull. No. 7 (1968).